# Basic Property of fluid

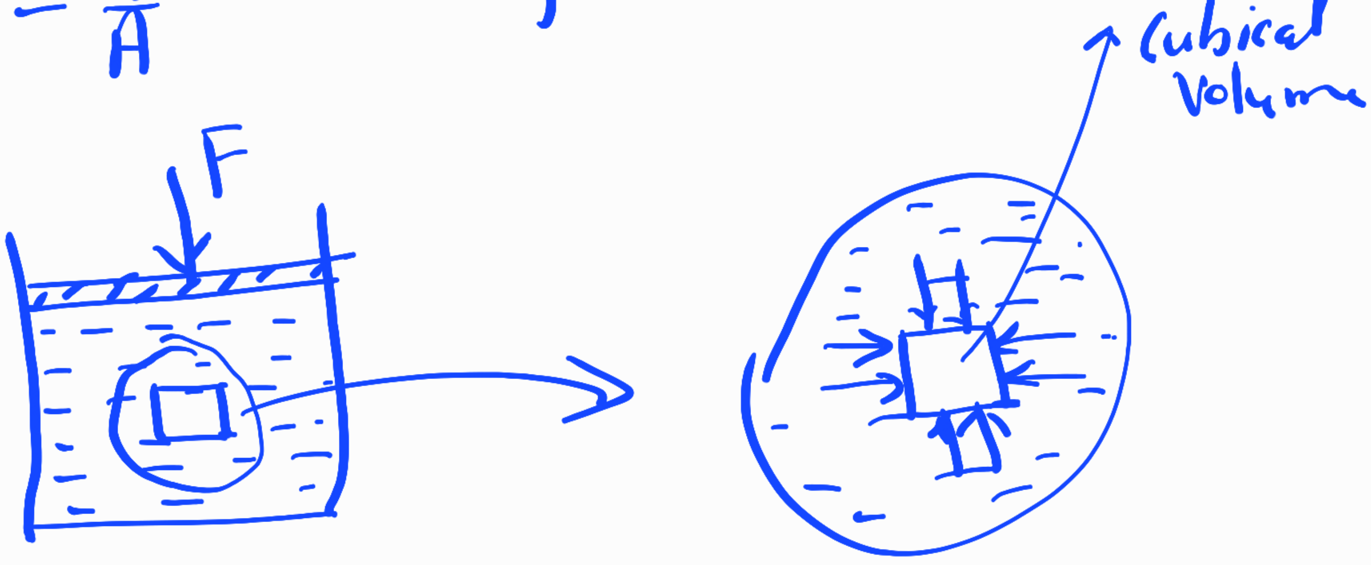
→ Definite Volume But no definit Shape

means same volume can aquire different types of shape.

→ Fluid don't support shear stress

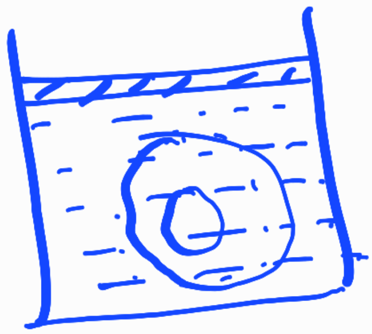
⇒ **The Concept of Pressure:**

$P = \frac{F}{A}$ → normal force

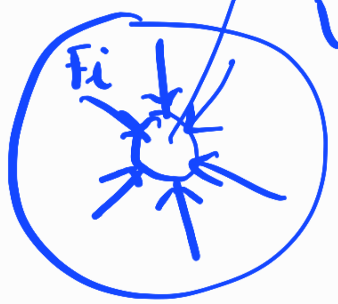



due to applied external force F

Some internal forces will be developed in the Static fluid and these will be normal to the bounding surfaces.



$F_i \rightarrow$ normal to the surface (hypothetical)



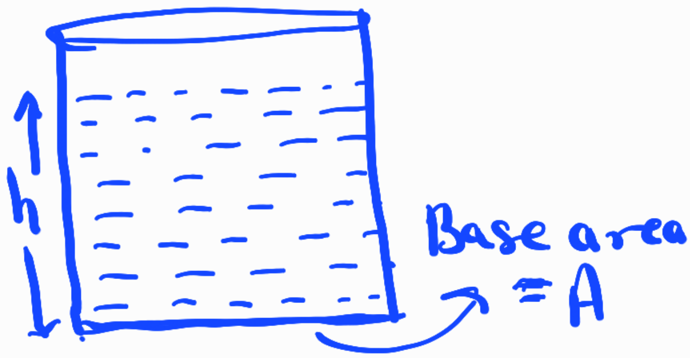

spherical volume

The above facts indicates that in fluids in the state of equilibrium there can only appear normal internal forces, and these forces always has a tendency to compress the bounded volume in the fluid.

That is why in fluid mechanics we talk about the pressure not about vector forces. This pressure in the fluid is the compressive forces active per unit area of the hypothetical surface and always normal to the surface

## ⇒ Pressure distribution in a static fluid:-

Consider the Container shown in figure. Liquid density $= \rho$




The weight of the liquid exerts the force on the bottom of the container

Here force on the bottom = weight of Liquid

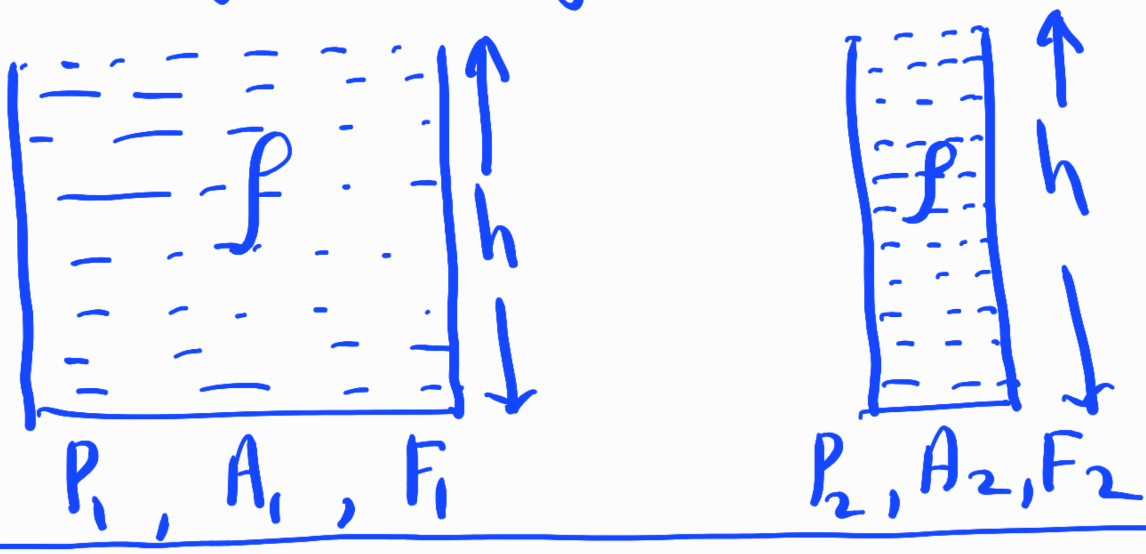
$$F = mg \quad , \quad m = Ah\rho$$

$$P = \frac{F}{A} = \frac{A\rho h g}{A} = \rho g h$$

hence P only depends on the $\rho$ and depth of Liquid and not the Shape and Cross sectional area of the Container.

But here we have to note that

total force on the bottom surface due to the Liquid is grater at the bottom of the Large container.




$$P_1 = \rho g h \quad = \quad P_2 = \rho g h$$

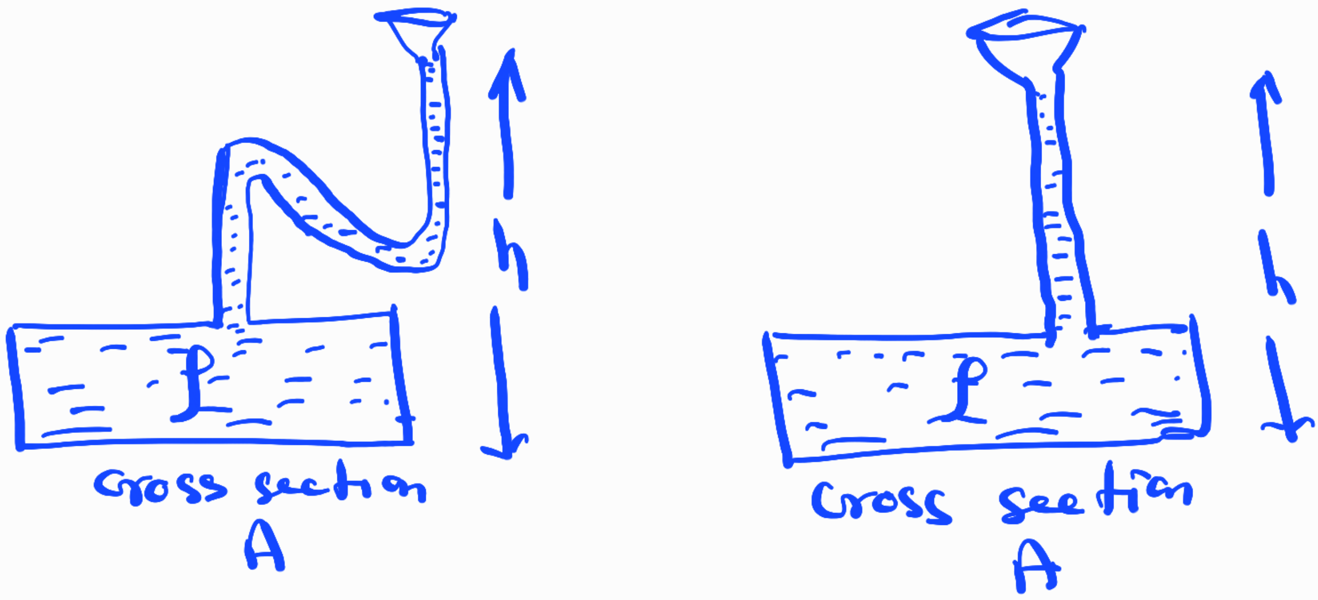
$$F_1 = P_1 A_1 \qquad\qquad F_2 = P_2 A_2$$
$$= \rho g h A_1 \qquad\qquad = \rho g h A_2$$

As $A_1 > A_2$

$F_1 > F_2$

Force on the Large container's bottom will be grater.

An another example:




Here

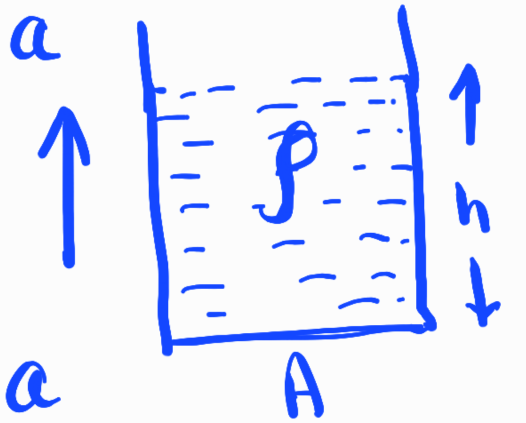
F at the bottom

$$F = \text{Pressure at bottom} \times \text{cross section area}$$

$$= \rho g h \times A = \rho g h A$$

we can observe that the cross section area is same in both situations then F at bottom and Pressure at bottom only depend on the height up to the free top surface of the liquid, no matter whatever be the amount of liquid present and whatever be the shape of the container up side.

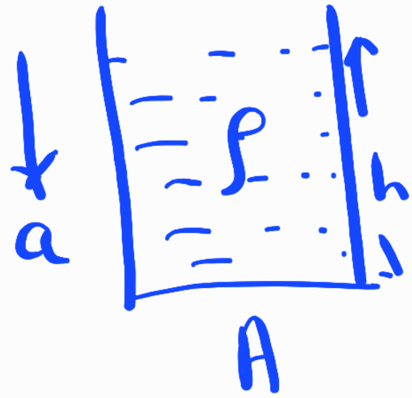
→ Consider the container moving up with acceleration $a$




Pressure on the bottom

$$P_1 = \rho h g_{eff} \,, \quad g_{eff} = (a+g)$$
$$= \rho h (a+g)$$

if moving downward with acceleration $a$ then

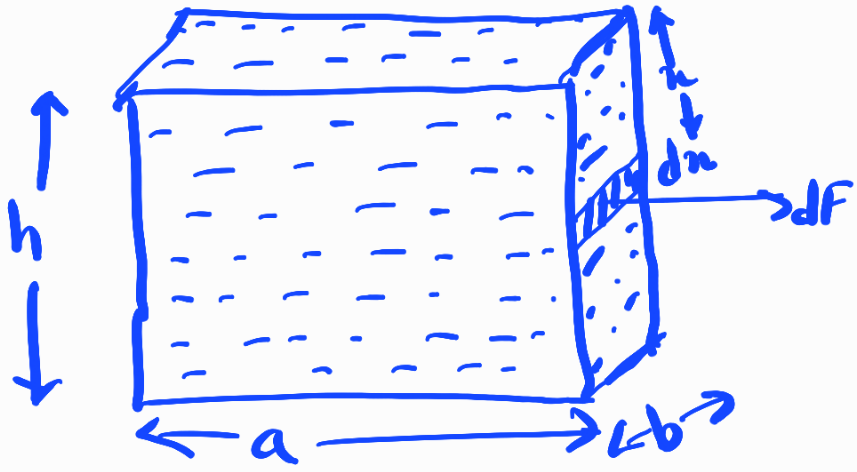



$$P_2 = \rho h g_{eff}$$
$$= \rho h (g-a) \,, \quad g_{eff} = (g-a)$$

here $P_1 > P_2$

## ⇒ Force on side wall of a vessel :

Force on side wall not be directly determind as Pressure is different at different height




$$dF = x\rho g \times b\,dx$$

$$F_b = \int_0^h x\rho g\, b\,dx = \frac{1}{2}\rho g h^2 b$$

Similarly $F_a = \frac{1}{2}\rho g h^2 a$

if $a > b$ then $F_a > F_b$

hence force on Larger side wall will be Larger.

here $F \propto h^2$

→ Average Pressure on the side wall:-

the absolute Pressure on the side wall can not be evaluated becase Pressure is different at different depth.

The average Pressure

$$\langle P \rangle_{av} = \frac{F}{\text{Area of side wall}} = \frac{F}{bh}$$

$$= \frac{1}{2} \rho g h$$

As it does not depend on the lenght of side.

and it shows that $\langle P \rangle_{av}$ on the side vertical wall is half of the net Pressure at the bottom of the vessel.

→ Torque on the side wall due to fluid Pressure:

Here, due to force $dF$, the side wall experience a torque about the bottom edge of the side, $d\tau$

$$d\tau = dF \times (h-x)$$

$$\tau = \int d\tau = \int_0^h x\rho g b\, dx\,(h-x)$$

$$= \int_0^h \rho g b\,(hx - x^2)\,dx$$

$$= \frac{1}{6}\rho g h^3 b$$

Here $\tau \propto h^3$

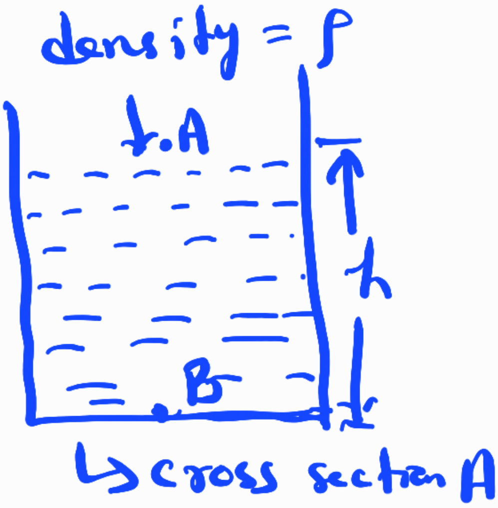
→ Atmospheric Pressure and effect of atmospheric pressure on the system:-

It is the pressure of the earth atmospheric. Normal atmospheric Pressure at sea level is 1 atm we can denot it as $P_0 = 1\ atm$
$= 1.013 \times 10^5\ Pa$
$= 1.013 \times 10^5\ N/m^2$

→ Fluid force act perpendicular to any Surface in the floor, no matter how that surface is oriented. hence Pressure has no intrinsic direction of its own. means it is a Scalar.

→ The excess pressure above the atmospheric pressure is called the gauge pressure and total Pressure is called the absolute Pressure.

Now consider the figure here when we take atmospheric Pressure in account




The Pressure at Point A

$$P_A = P_0 \longrightarrow \text{atmospheric pressure}$$

The Pressure at Point B

$$P_B = P_A + \rho g h = P_0 + \rho g h$$

hence, Gauge pressure is $\rho g h$

and absolute Pressure is $P_0 + \rho g h$

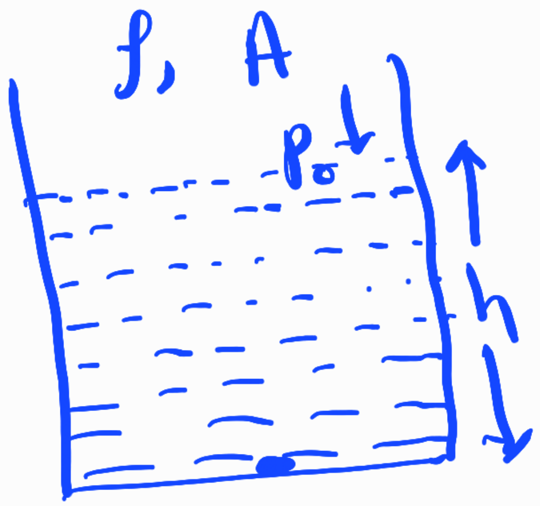
we can also caculate the variation of Pressure w.r.t depth $h$

here Pressure at depth $h$ from liquid surface, $P = P_0 + \rho g h$

$$\frac{dP}{dh} = \rho g$$ → This is called the Pressure gradient

(+)ve sigh shows that Pressure increases with depth from the surface of the liquid.

→ Now when we are going to solve the Problems we should keep the following facts in mind.



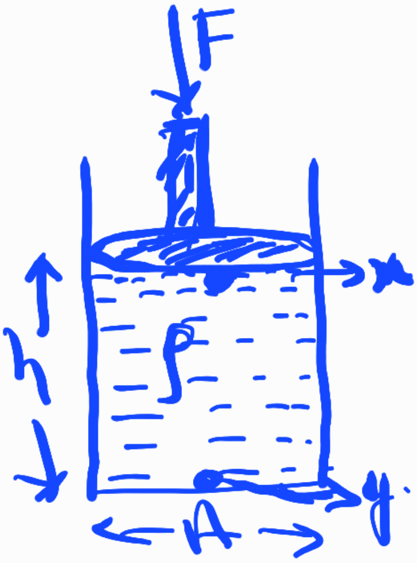

1.) density $\rho$,
Crosssection A
and if it is mentioned that "an open container" that means Pressure at the surface of the fluid is atmospheric Pressure.

The Pressure at the bottom of the vessel will be $P = P_0 + \rho g h$.

2.) when "container is clossed" that means no atmospheric pressure on the surface, then pressure at the bottom

$$P = \rho g h$$

3.) Consider the situation where a Piston kept on the surface of Liquid and an external force F is applied as shown in figure.




The Pressure at the Point on the fluid surface just below the Piston

$$P_x = \frac{F}{A}$$ , $$P_y = \frac{F}{A} + \rho g h$$

↳ at the bottom

Here we should note that if nothing mentioned about the mass of the Piston then we should consider it as massless Piston.

Now suppose no external force is applied and the Piston is massless then

$$P_a = P_0 \text{ (atmospheric Pressure)}$$

$$P_y = P_0 + \rho g h$$

→ Here we should ve very cleare that Piston is free to move so the Pressure on it is atmopheric Pressure and we shold not be confused with the Closed Container case.

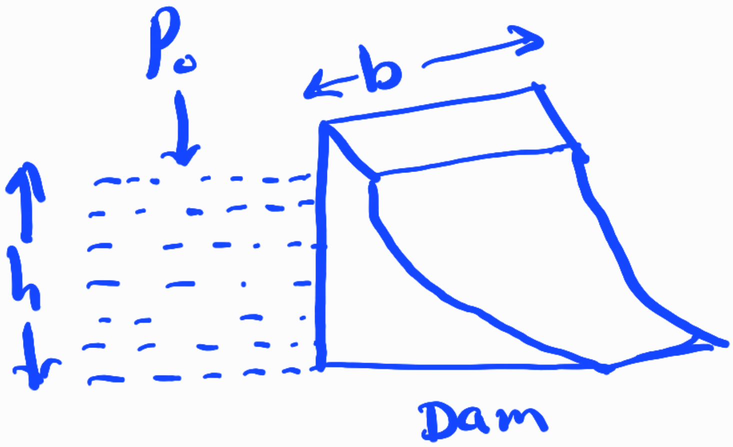
Now consider the same Piston is of mass m and there is no external force is applied and no atmospheric Pressure. then,

the Pressure on the surface will be due to the weight of the Piston

$$P_x = \frac{mg}{A}$$

$$P_y = \frac{mg}{A} + \rho g h \quad \text{(at bottom)}$$

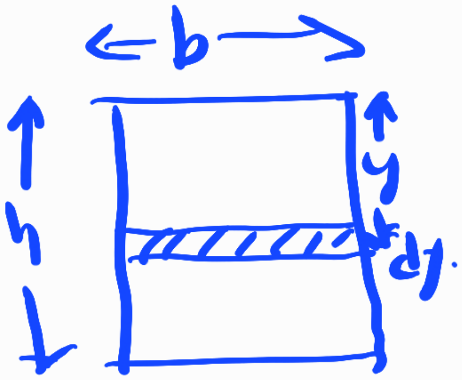
Problem:




Find the force excerted by a Liquid of density $\rho$ on the dam of width b as shown in figure. The atmospheric pressure is $P_0$.

Sol:

Consider the differential area of width dy and lenght b at a depth y from the free surface of the Liquid.




$$dF = (P_0 + \rho g y)\, b\, dy$$

$$F = \int_0^h dF = \int_0^h (P_0 + \rho g y)\, b\, dy$$

$$F = P_0 b h + \frac{1}{2}\rho g b h^2$$

This force has two part

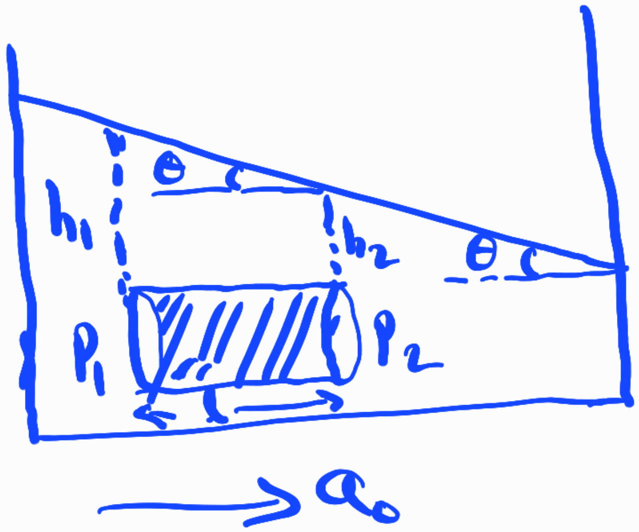
force due to atmospheric Pressure $= P_0 b h$

force due to depth (varying) of Liquid $= \frac{1}{2}\rho g b h^2$

⟹ Variation of Pressure along horizontal in accelerating fluids :-

(i)

If the Container is open at the top. The shape of the liquid will be changes as shown in the figure.




The shaded element acceleration to the right


$$F_1 - F_2 = m a_0$$

$$F_1 = P_1 \Delta A \,, \qquad P_1 = P_0 + \rho g h_1$$

$$F_2 = P_2 \Delta A \,, \qquad P_2 = P_0 + \rho g h_2$$

$$m = \Delta A \, l \, \rho$$

$$P_1 \Delta A - P_2 \Delta A = \Delta A l \rho a_0$$

or, $\rho g (h_1 - h_2) \Delta A = \Delta A \, l \rho a_0$

$$\Rightarrow \quad \frac{h_1 - h_2}{l} = \frac{a_0}{g}$$

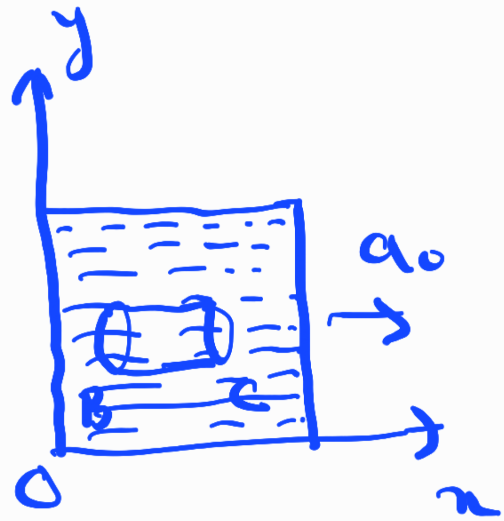
From the figure, $\tan\theta = \frac{h_1 - h_2}{l}$

$$\Rightarrow \quad \tan\theta = \frac{a_0}{g}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a_0}{g}\right)$$

(ii) if the container is completely filled and closed, the pressure varies vertically as well as horizontaly.

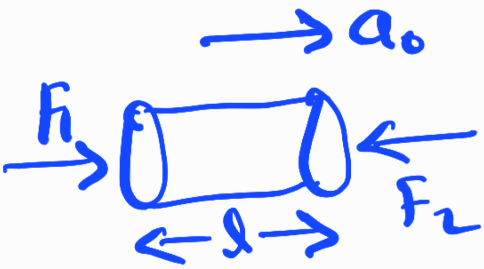
As we already know the rate of change of Pressure from Surface to the bottom




$$\frac{dP}{dy} = \rho g$$

hence from the bottom to surface i.e. along oy is $\frac{dP}{dy} = -\rho g$.

$$F_1 - F_2 = ma_o$$




$$F_2 - F_1 = -ma_o$$

$$P_2 \Delta A - P_1 \Delta A = -\Delta A l \rho a_o$$

$$\frac{P_2 - P_1}{l} = -\rho a_o$$

$$\Rightarrow \frac{dP}{dx} = -\rho a_o$$

means Pressure decreases while moving in the direction of $a_o$.